**Title**

Scaling of oscillatory kinematics and Froude efficiency in baleen whales

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**Running Title**

Scaling of Cetacean Swimming Efficiency

**Summary Statement**

We used tags attached to baleen whales to demonstrate how thrust power output, drag coefficient, and Froude efficiency scale with swimming speed and body length.

**Abstract**

High efficiency lunate-tail swimming with high-aspect-ratio lifting surfaces has evolved in many vertebrate lineages, from fish to cetaceans. Baleen whales (Mysticeti) are the largest swimming animals that exhibit this locomotor strategy and present an ideal study system to examine how morphology and the kinematics of swimming scale to the largest body sizes. We used data from whale-borne inertial sensors coupled with morphometric measurements from aerial drones to calculate the hydrodynamic performance of oscillatory swimming in six baleen whale species (fin whale, *Balaenoptera physalus*; Bryde’s whale, *Balaenoptera* edeni; sei whale, *Balaenoptera* borealis; Antarctic minke whales, *Balaenoptera bonaerensis*; humpback whales, *Megaptera novaeangliae*; and blue whales, *Balaenoptera musculus*) ranging in body length from 5-25m. We find that mass-specific thrust increases with both swimming speed and body size. Froude efficiency, defined as the amount of mechanical work the animal does to propel itself forward, increased with swimming speed to a plateau but decreased with body size. This finding is contrary to previous data for smaller animals that Froude efficiency is positively related to body size. Although our empirically parameterized estimates for the drag of a swimming baleen whale was higher than a simple gliding model, oscillatory locomotion at this scale exhibits high Froude efficiency as in other adept swimmers. Our results quantify the fine-scale kinematics and hydrodynamics of routine and energetically expensive swimming modes at the largest scale.

**Introduction**

The repeated invasion of aquatic and marine environments by tetrapods over the last 250 million years has resulted in a host of convergent morphological adaptations that facilitate life in water (Kelley & Pyenson, 2015). Among these adaptations are the evolution of a fusiform body shape, flattened control surfaces, and sickle-shaped caudal fin to achieve high performance locomotion (Fish et al., 2008). These morphological adaptations are functionally analogous with other swimming animals such as thunniform fish, lamnid sharks, cetaceans, and the extinct ichthyosaurs (Motani, 2002; Donley et al., 2004; Gleiss et al., 2011). The majority of these swimmers use an oscillatory swimming style that involves side-to-side or up-and-down movement of a hydrofoil-like tail to generate lift-based thrust and overcome drag (Fish, 1998). Cetaceans are unique among oscillatory swimmers because of their extreme body mass, exemplified in modern baleen whales (Mysticeti), which evolved massive body sizes within the last five million years (Slater et al., 2017).

Although the swimming performance of large whales has long been of interest to researchers (Krogh, 1934; Kermack, 1948; Bose & Lien, 1989), direct measures of their swimming kinematics and morphology have been difficult to obtain. Studies of cetacean swimming kinematics have typically focused on smaller and highly maneuverable odontocete species in captivity (Fish, 1993; Curren et al., 1994; Fish, 1998). Attempts to study mysticetes have been constrained to breathing events at the water’s surface. Energetic assumptions (Sumich, 1983; Parry, 1949; Blix & Folkow, 1995) and morphological measurements were limited to deceased animals that had stranded on beaches or been captured by whaling operations (Lockyer, 1976; Kahane-Rapport & Goldbogen, 2018). The recent development of high-resolution biologging methods now allows researchers to quantify the kinematics of free-swimming cetaceans in their natural habitats (Johnson & Tyack, 2003; Goldbogen et al., 2017a; Gough et al., 2019). In addition, unoccupied aircraft systems (UAS, or drone) technology has enhanced our ability to obtain precise morphological data, thereby enabling comparative and scaling analyses of form and function (Kahane-Rapport et al., 2020; Gough et al., 2019).

Understanding the size-dependent kinematics of swimming cetaceans is critical to analyze their swimming performance and energetics. The dorso-ventral oscillation of the flukes produces lift that is resolved into a forward thrust vector (Fig. 1; Lighthill, 1971; Chopra and Kambe, 1977; Vogel, 1994; Fish, 1998). This lift-based thrust power is equal to the drag power of the animal when swimming at a constant velocity (Lighthill, 1971; Fish, 1998). This mechanism is considered to be highly efficient (>75%; Triantafyllou et al., 1991; Rohr & Fish, 2004). Previous attempts to estimate the thrust power of actively swimming large whales have been made based on a number of assumptions without reliable kinematic data (Parry, 1949; Chopra & Kambe, 1977; Yates, 1983; Bose & Lien, 1989). Thrust power generation is modulated through the adjustment of basic kinematic parameters of the oscillatory tailbeat cycle, and new biologging tags make these empirical measurements possible for large, free-swimming animals.

Kinematic studies performed on cetaceans have focused on the three fundamental parameters of an oscillatory tailbeat cycle: amplitude of heave, swimming speed, and oscillatory frequency. Among these, speed has been studied most extensively. Using various methods, researchers have found that many different species of cetaceans are able to swim over an extended range of speeds. High speeds in excess of 8 m/s have been achieved by rorqual mysticetes (Fish & Rohr, 1999; Hirt et al., 2017; Segre et al., 2020). A recent study by Gough et al. (2019) has shown that mysticetes tend to swim at ~2 m/s when they are not feeding. In order to swim at different speeds within this wide range, mysticetes must adjust either their oscillatory frequency or the amplitude of heave (Lighthill, 1971; Chopra & Kambe, 1977). For small odontocetes, Fish (1998) found that oscillatory frequency increased with increasing swimming speed but decreased roughly with body length while amplitude of heave remained constant at ~0.2 of an animal’s body length. These findings were recently confirmed for mysticetes by Gough et al. (2019).

Measuring the fundamental kinematic parameters of the oscillatory tailbeat cycle has allowed researchers to estimate Froude efficiency, or the percentage of thrust that is successfully transferred into forward motion (Vogel, 1994; Fish, 1998). The dimensionless Strouhal number has typically been used as a rough way to describe how the amplitude of heave, swimming speed, and oscillatory frequency are modulated and interact to provide a maximally efficient pattern of vorticity around the tail during swimming (Triantafyllou et al., 1991; Fish, 1998; Taylor et al., 2003; Rohr & Fish, 2004; Gough et al., 2019). The generally accepted rule is that highly-efficient oscillatory swimming falls within a Strouhal range from 0.25-0.35 (Triantafyllou et al., 1991). Both Rohr & Fish (2004) and Gough et al. (2019) found that cetaceans fall within this range, but a more detailed analysis of the kinematics and hydrodynamic parameters, such as the thrust power output and drag, has only been performed previously by Fish (1998) for much smaller odontocetes.

Here, our goal is to move beyond the Strouhal number and use a combination of whale-borne tags and UAS morphological measurements to calculate the kinematics, thrust power output, and Froude efficiencies for free-swimming mysticete whales using similar methods to Fish (1998). Apart from Gough et al. (2019), we have a very limited understanding of how kinematics affect swimming performance at the upper extremes of body size. Previous studies have estimated the Froude efficiency of swimming for odontocetes and other oscillatory swimming animals to be approximately ~75-90% (Fish, 1998), but the only estimate for a mysticete before our study came from a single fin whale (*Balaenoptera physalus*) of unknown body size swimming at ~8 m/s (Bose & Lien, 1989). Our current data set goes beyond any previous analyses and includes six species and a ~5x range in body length. All of the species included in our study are lunge feeders that feed by opening their mouth and engulfing a large volume of water into a highly expansible throat pouch (Goldbogen et al., 2017b). This behavior requires the efficient achievement of high swimming speeds in order to maintain a favorable energetic balance (Potvin et al., 2009). We hypothesize that the kinematic and hydrodynamic parameters of swimming scale similarly between small and large cetaceans and will lead to high (>75%) Froude efficiencies for even the largest animals. Our study will lead to a more complete scaling-based understanding of oscillatory swimming in mysticetes and the kinematic, hydrodynamic, and morphological factors that impact swimming performance in the world’s largest animals.

**Methods**

*Study species and locations*

The whales included in this study are the Antarctic minke whale (*Balaenoptera bonaerensis*, Burmeister, 1867), humpback whale (*Megaptera novaeangliae*, Borowski, 1781), fin whale (*Balaenoptera physalus*, Linnaeus, 1758), Bryde’s whale (*Balaenoptera* edeni, Anderson, 1879), sei whale (*Balaenoptera borealis*, Lesson, 1828), and blue whale (*Balaenoptera musculus*, Linnaeus, 1758). The six species are members of the family Baleanoptera, commonly referred to as rorquals, and tend to have similar life histories and behaviors. These species range in size from ~5 m in length for the Antarctic minke whale up to ~25 m for an adult blue whale (Goldbogen et al., 2019). Distinct morphological differences are also present between these species, with the most prominent being the enlarged flukes and flippers of the humpback whale relative to body size (Fish & Battle, 1995; Woodward et al., 2006).

Data on foraging and swimming was collected on humpback whales off of the coast of Monterey, CA and the Western Antarctic Peninsula, blue whales off California (Monterey Bay and Southern California Bight), Antarctic minke whales off the western Antarctic Peninsula, fin whales in Monterey Bay and the fjords of southeastern Greenland, Bryde’s whales off the southern coast of South Africa, and sei whales near the Falkland Islands**.** All work was performed under suitable permits and in accordance with university IACUC procedures (See Acknowledgements section below).

*CATS Tags*

The Customized Animal Tracking Solutions (CATS) tags integrate video with 400 Hz accelerometers and gyroscopes; 50 Hz magnetometers, pressure and temperature sensors; a 10 Hz internal temperature sensor; and 10 Hz light and GPS sensors. Tag accelerometers for all whales were sampled at 40 or 400 Hz, magnetometers and gyroscopes at 40 or 50 Hz, and pressure, light, temperature, and GPS at 10 Hz. All data were decimated to 10 Hz, tag orientation on the animal was corrected for, and animal orientation was calculated using custom-written scripts in Matla 2014a (following Johnson & Tyack, 2003; Cade et al., 2016). Animal speed for all deployments was determined using the amplitude of tag vibrations (Cade et al., 2018). Tags were deployed from rigid-hull inflatable boats using a 6 m carbon-fiber pole. Tags attached to the animal with four suction cups, detached after suction failed, floated to the surface and were recovered via VHF telemetry. Deployment lengths in this study ranged from 8 mins to 26 hrs. For more information on the tags used in this study, see Goldbogen et al. (2017a).

UAS Operations and *Morphometric Measurements*

Images of each species were collected using UAS between 2017 and 2019. Specifically, two types of stock-build quadcopters, the Phantom 3 and Phantom 4 Pro, as well as two types of custom hexacopters were used, the FreeFly Alta 6 and a Mikrokopter-based LemHex-44. Both quadcopters used stock-built barometers and cameras while the hexacopters contained a 2-axis gimbal fitted with a Lightware SF11/C laser altimeter and a Sony Alpha A5100 camera with an APS-C sensor (23.5 mm by 15.6 mm), 6000 x 4000 pixel resolution, and either a Sony SEL 50 mm or SEL 35 mm focal length low distortion lens.

ImageJ 1.5i (Schindelin et al., 2012) was used to measure the total length, maximum body diameter, fluke chord length, and fluke area (Fig. 2). Measurement errors for each aircraft were estimated by measuring a known sized object floating at the surface from various altitudes, and each aircraft had an average error < 5%. Measurements in pixels were multiplied by the ground sampling distance (GSD) to convert to meters following Fearnbach et al. (2012):

(1)

(2)

where *L is the length (m), npix* is the number of pixels, *a* is the altitude, *lfoc* is the focal length (mm), *Sw* is the sensor width (mm), and *Pw* is the image resolution width (px). The width of the sensor and image resolution was used since images of the whales were captured full frame widthwise (Gough et al., 2019). In ImageJ (NIH), the combined planar surface area of the flukes (*Fa*; m2) was calculated by carefully drawing a polygonal outline of the flukes. Chord length of the flukes (C; m) was measured as the linear distance from the notch between the flukes to the anterior insertion of the flukes on the tail. Body mass (kg) was estimated from total body length using known regressions (Kahane-Rapport & Goldbogen, 2018). The wetted surface area of the body (*Sa* ;m2) was estimated from total body length using data provided by Fish (1993).

*Thrust Power, Efficiency, and Drag Coefficient Modeling*

We used a customized MATLAB script to detect tailbeat cycles based upon methods defined by Gough et al. (2019). In particular, a series of thresholds were used to define periods in the filtered (low-pass) gyroscope signal (along the transverse axis) corresponding to individual tailbeats. These thresholds checked for symmetricality between the upstroke and downstroke by defining the magnitude, duration, and overall shape of each portion of the tailbeat cycle. The resulting set of tailbeat cycles was spot-checked and compared against tag video to ensure that the parameters were set correctly. Individual whales must have had a dataset of >200 tailbeats in order to be included for further analysis.

For each tailbeat, we measured the mean swimming velocity (*U*; m s-1) by averaging across the entire time course of the cycle. Since the measurement of speed by the tag requires turbulent flow, we were limited to only measure speeds >1 m s-1 (Cade et al., 2018). We also measured oscillatory frequency (*f*; Hz) as the inverse of the duration of the cycle. Our calculations of thrust power (*PT*), coefficient of drag (*CD*), and Froude efficiency (*η*) were based on a model of lunate tail propulsion using unsteady wing lifting surface theory (Chopra & Kambe, 1977; Yates, 1983; Fish, 1998). This model begins with the estimation of two input parameters, namely the reduced frequency (*σ*) defined as:

(3)

where *ω* is the angular frequency of fluking (with *ω =* 2π*f*); and the feathering parameter (*θ*) defined as:

(4)

which is expressed as the ratio of the maximum angle (*α*, in degrees) between the fluke and the direction of motion and the maximum angle (*ωh1/U*) achieved by the trajectory of the pitching axis of the flukes (Yates, 1983) when reaching the heave amplitude *h1 (m)*. We were unable to measure precise values for the *α* or *h1* from the tag data and instead relied on validated estimates of 30° for *α* and one-fifth of body lengh for *h1* (Bainbridge, 1958; Fish, 1998).

The model devised by Chopra & Kambe (1977) yielded a series of parametric curves expressing *CT* and *η* in terms of and (Yates, 1983). We digitized these curves and estimated both values for each tailbeat cycle. We then estimated the mean thrust force () (over a tailbeat cycle),given the relationship:

(5)

where is the density of seawater, is the fluke area, and *c* is the chord length of the tail from insertion to tail tip. The (mechanical) thrust power output (*PT*) is given by:

(6)

and equating the mechanical power generated with the drag power dissipated by the body yields the (mean) drag force and its corresponding drag coefficient: (Fish, 1993):

(7)

*Routine vs. Lunge-Associated Swimming*

Foraging lunges were detected manually using a series of defined parameters that have been validated using tag video (Cade et al., 2016). These events typically involve an increase in speed followed by a rapid deceleration as the animals opens its mouth (Potvin et al., 2009; Goldbogen et al., 2011; Cade et al., 2016). We chose to define the period from 10-0 seconds prior to the lunge deceleration as the lunge-associated period and any tailbeat that occurred within that time was similarly classified as lunge-associated. All other tailbeats were classified as routine swimming. The lunge-associated tailbeats likely included a greater change in swimming velocity, but our tailbeat detection thresholds ensured general consistency in the overall kinematic profile of the tailbeats and resulted in two sets of tailbeats at different levels of swimming effort.

*Comparison to a Simple Rigid-Body Model*

Cetacean swimming involves body and tail heaving motions that are altogether absent with the motions of rigid bodies (e.g., submarines) and significantly increase drag (Fish, 1993; Fish, 1998; Fish & Rohr 1999). We compared our drag coefficient data with that of airship models tested in wind tunnels, as summarized in the following correlation (Hoerner, 1965; Blevins, 1984; Webb, 1975; Kooyman, 1989):

(8)

where is the maximum body diameter (m). This equation is expressed in terms of the Reynold’s number:

(6)

in which is the kinematic viscosity. In this case the drag force () sustained by the airship (or non-tail-heaving whale) is given by:

(10)

*Statistical Analyses*

We compared hydrodynamic performance variables (thrust power output, drag coefficient, and Froude efficiency) with swimming speed, Reynolds number and morphometric measurements using R v. 3.6 and RStudio (Version 1.2.1335, packages: ggpubr, and tidyverse) (R Core Team, 2014; Wickham et al., 2019; Kassambara, 2020). We fit linear models to assess relationships using package lm in R. For our analysis of swimming speed vs Froude efficiency, we used a generalized additive model (GAM) in R (y ~ s[x, bs = “cs”]).

**Results**

*Kinematic and Morphometric Summary*

We investigated interspecific relationships between 68 animals and found that mean (± se) values for oscillatory frequency (Hz) and swimming speed (m s-1) both increased when transitioning from routine to lunge-associated swimming. The mean increase in swimming speed between the two modes was 0.695 ± 0.152 m s-1 and the mean increase in oscillatory frequency was 0.09 ± 0.02 Hz.

We found that the mean oscillatory frequency for the three species with the most data (humpback, blue, Antarctic minke) decreased with increasing body length with the Antarctic minke whale having the highest values (routine: 0.40 ± 0.010 Hz; lunge-associated: 0.49 ± 0.008 Hz), followed by the humpback whale (routine: 0.24 ± 0.006 Hz; lunge-associated: 0.34 ± 0.011 Hz) and the blue whale (routine: 0.19 ± 0.004 Hz; lunge-associated: 0.24 ± 0.004 Hz). We found that Bryde’s and fin whales had similar routine oscillatory frequencies as the humpback whale while having longer average body lengths (Bryde’s: 12.04 ± 2.07 m; fin: 18.90 ± 0.43 m) than the humpback whales in our study (11.09 ± 0.33 m). Both of the oscillatory frequency values for the lone tagged sei whale (routine: 0.22 Hz; lunge-associated: 0.30 Hz) fell approximately halfway between the values for the humpback and blue whales, which aligns with the sei whale’s body length (16.62 m) being approximately halfway between the mean humpback and blue whale (22.50 ± 0.32 m) body lengths. We found that the oscillatory frequency decreases as the total length increases. We found significant negative relationships between oscillatory frequency and body size during both routine and lunge-associated swimming (routine: *ŷ* = -0.560x + 0.313; *R2* = 0.77; *p* < 0.001; lunge-associated: *ŷ* = -0.565x + 0.009; R2 = 0.76; p < 0.001; Fig. 3A).

The mean values for both routine and lunge-associated swimming speeds were similar for the humpback (routine: 2.15 ± 0.066 m s-1; lunge-associated: 2.85 ± 0.100 m s-1), blue (routine: 2.27 ± 0.063 m s-1; lunge-associated: 3.05 ± 0.056 m s-1), and Antarctic minke whales (routine: 2.44 ± 0.053 m s-1; lunge-associated: 2.96 ± 0.117 m s-1). Despite low sample sizes, the average routine and lunge-associated swimming speeds for the Bryde’s whale (routine: 1.76 ± 0.51 m s-1; lunge-associated: 3.11 ± 0.629 m s-1) and the routine swimming speed for the sei whale (2.23 m s-1) aligned with the humpback, blue, and Antarctic minke whales, while the lunge-associated swimming speed for the sei whale (2.46 m s-1) was lower than other values and both swimming speeds were higher for the fin whale (routine: 3.02 ± 0.125 m s-1; lunge-associated: 3.61 ± 0.900 m s-1). We found that swim speed stays fairly constant as the total length increases. Our statistical analysis found no effect of body size on swim speed for both routine and lunge-associated swimming (routine: *ŷ* = -0.003x + 0.785; *R2* = 6.25\*10-5 ; *p* = 0.949; lunge-associated: *ŷ* = 0.078x + 0.873; R2 = 0.04; p = 0.098; Fig. 3B). All species-level means (± se) for each of our measured kinematic and morphometric variables are given in Table 1. The equations and statistics pertaining to our models are given in Table 2.

*Mass-Specific Thrust Power Output*

Among the three species with a large amount of data in our dataset (humpback, blue, and Antarctic minke whales), the humpback whale had the lowest mean mass-specific thrust power output (0.30 ± 0.023 Watts kg-1), with the Antarctic minke whale having a slightly higher value (0.36 ± 0.028 Watts kg-1) and the blue whale having the highest value (0.48 ± 0.025 Watts kg-1). The Bryde’s (0.50 ± 0.213 Watts kg-1), sei (0.51), and fin whale (0.74 ± 0.130 Watts kg-1) each had higher values.

We found that mean mass-specific thrust power output increased with the transition from routine to lunge-associated swimming modes (Fig. 4A-B). There was a positive effect of swimming speed on mass-specific thrust power output during both routine and lunge-associated swimming (routine: *ŷ* = 731x – 2.764; *R2* = 0.40; *p* < 0.001; lunge-associated: *ŷ* = 0.630x – 2.138; R2 = 0.49; p < 0.001; Fig 4A). We also found that mean mass-specific thrust power output increases with body length for both routine (*ŷ* = 0.033x – 1.594; *R2* = 0.23; *p* < 0.001) and lunge-associated swimming (*ŷ* = 0.031x- 0.693; *R2* = 0.16; *p* < 0.001; Fig. 4B). The species-level means (± se) for each of our measured hydrodynamic parameters are given in Table 3.

*Drag Coefficient*

Among humpback, blue, and Antarctic minke whales, the Antarctic minke whale had the lowest mean drag coefficient (0.01 ± 0.001), with the humpback whale slightly higher (0.02 ± 0.002) and the blue whale having the highest value (0.04 ± 0.006). We found that the drag coefficient for both routine and lunge-associated swimming decreased with increasing swim speed (routine: *ŷ* = - 0.613x – 2.617; *R2* = 0.12; *p* = 0.004; lunge-associated: *ŷ* = -0.270x – 3.148; R2 = 0.06; p = 0.042; Fig 5A). Conversely, we found that the drag coefficient increased for both routine and lunge-associated swimming with increasing total body length (routine: *ŷ* = 0.082x – 5.080; *R2* = 0.64; *p* < 0.001; lunge-associated: *ŷ* = 0.058x – 4.737; *R2* = 0.41; *p* < 0.001; Fig 5B).

We found that the drag coefficient increased significantly with Reynolds number (routine: *ŷ* = 2.92\*10-8x – 4.805; *R2* = 0.41; *p* < 0.001; lunge-associated: *ŷ* = 1.36\*10-8x – 4.476; *R2* = 0.23; *p* < 0.001; Fig 5C). In comparison to the R-100 rigid-hulled airship model, all species displayed higher drag coefficients for both swimming modes (Fig 5C).

*Froude Efficiency*

Of the three species with a large quantity of data in our dataset (humpback, blue, and Antarctic minke whales), the Antarctic minke whale had the highest mean Froude efficiency (0.920 ± 0.004), with the humpback having a lower mean value (0.908 ± 0.003) and the blue whale having the lowest mean value (0.860 ± 0.006). The mean values for the Bryde’s (0.868 ± 0.022), sei (0.876), and fin whales (0.889 ± 0.018) were all near the low end of the range.

We found that mean Froude efficiency increases with increasing swimming speed up to a plateau at ~3 m s-1 and began to drop off at 5 m s-1 for lunge-associated tailbeats (Fig. 6A). On the other hand, we found that mean Froude efficiency decreased with increasing body length (routine: *ŷ* = -0.004x – 0.049; *R2* = 0.68; *p* < 0.001; lunge-associated: *ŷ* = -0.003x – 0.068; *R2* = 0.47; *p* < 0.001; Fig. 6B). As compared to prior studies, our results demonstrate that, regardless of body size, rorqual whales demonstrate high efficiency (>75%) comparable to other oscillatory swimmers (Fig. 7). Sub-carangiform, undulatory swimmers such as the rainbow trout (*Oncorhynchus mykiss*) are slightly lower (~60-80%) and drag-based swimmers, such as the muskrat and human, have much lower Froude efficiencies (~20-35%) (Fig. 7). Table 4 gives additional information about each literature-based mean Froude efficiency value.

**Discussion:**

Many previous studies that have quantified the kinematics and hydrodynamics of cetacean swimming have used captive animals that can be measured reliably from a stable reference position (Fish, 1993; Fish, 1998; Rohr & Fish, 2004). By comparison, the present study is a first approximation for many of the same kinematic variables of much larger species in their natural environment. Several parameters, such as the angle of attack of the flukes relative to the body or the amplitude of heave are still generally unknown (except in rare circumstances, see Gough et al., 2019), so we supplemented our empirical data with validated estimates for these unknown variables (Bainbridge, 1958; Fish, 1998). The angle of attack of the fluke has been found to change with speed over a range from 20-40°, so we used 30° as an average value (Fish, 1998). Amplitude of heave has been reliably measured as one-fifth of body length and remains constant across swimming speeds and body size (Bainbridge, 1958; Fish, 1998). Our combination of empirical measurements and reliable estimates allowed us to quantify hydrodynamic and kinematic aspects of mysticete swimming using a numerical computation based on unsteady lifting-surface theory and derived by Chopra & Kambe (1977), which has also been validated for odontocetes by Fish (1998). The similarity between our methods and those of previous studies extends our ability to compare swimming performance across vast body size ranges.

*Oscillatory Frequency and Swimming Speed*

Our results illustrate that the transition from routine to lunge-associated swimming predictably results in increased oscillatory frequencies and swimming speeds as the animal prepares for a lunge (Fig. 3) (Goldbogen et al., 2011; Cade et al., 2016). Gough et al. (2019) found that the oscillatory frequency decreases with increasing body size to the power of -0.53, and with a more robust data set we have found a similar scaling exponent of -0.48. For swimming speed, we again found similar results to Gough et al. (2019) with swimming speed remaining consistent at ~2 m s-1. For both oscillatory frequency and swimming speed, the scaling exponents for routine and lunge-associated swim efforts were nearly identical, with a difference of 0.006 for oscillatory frequency and a difference of 0.001 for swimming speed. This suggests that, regardless of body size, mysticetes prepare for a feeding lunge through similar kinematic pathways which include a consistent increase in both oscillatory frequency and swimming speed. These results for oscillatory frequency and swimming speed align with previous results for fish and odontocetes that have shown that swimming speed is heavily modulated by oscillatory frequency (Bainbridge, 1958; Fish, 1998; Gough et al., 2019).

*Mean Mass-Specific Thrust*

Thrust generation is a fundamental aspect of any swimming mode and the achievable thrust for a swimming animal has a direct impact on its maximum swimming speed and, subsequently, the types and quantities of prey that it can capture (Fish, 1998; Potvin et al., 2009; Cade et al., 2020). Hydrodynamic theory states that thrust should increase with the square of velocity (Webb, 1975; Vogel, 1994). Thrust from an oscillating hydrofoil will further increase the thrust of a system by 3-to-5 times (Lighthill, 1971; Liu et al., 1997; Anderson et al., 2001; Fish et al., 2014). While this theory holds for animals of similar sizes, we found it advantageous to measure the mass-specific thrust to make comparisons between mysticetes and other cetaceans that vary across a wide range of body sizes.

For cetaceans, high mass-specific thrust allows odontocetes to capture fast-moving, individual fish (Maresh et al., 2004) and allows mysticetes to achieve high speeds during feeding lunges to offset the deceleration during prey engulfment as well as the potential escape response of different prey types (Cade et al., 2016; Cade et al., 2020). Fish (1998) measured the mass-specific thrust of odontocete species and found maximum mass-specific thrust values of 22.5 and 23.7 W kg-1 for *Pseudorca crassidens* and *Tursiops truncatus*, respectively. The maximum mass-specific thrust value for a mysticete (Bryde’s) swimming at 5.7 m s-1 in our study was found to be 19.7 W kg-1, but mass-specific thrust values at the species-level averaged between 0.30-0.75 W kg-1, which was two orders of magnitude lower (Fig 4A; Table 3). These results suggest that mysticetes typically maintain low average mass-specific thrust values in accordance with their relatively steady swimming speeds (~1.5-2.5 m s-1), but that they can attain extremely high mass-specific thrust power output when properly motivated. Swimming speeds higher than those found in our dataset have also been found for humpbacks (up to ~9 m s-1; Tomilin, 1957; Segre et al., 2020), indicating that they could be producing mass-specific thrust values on par with odontocetes during fast maneuvers such as surface breaches.

Our comparisons of speed-matched mass-specific thrust output between routine swimming and lunges suggest that whales likely alter oscillatory frequency in order to generate greater thrust during feeding (Gough et al., 2019). Mass-specific thrust at a routine swimming speed (~1.5-2.5 m s-1) results in a low propulsive energy cost (Gough et al., 2019). The similarity of the mass-specific thrust increase (~two-fold) from routine to a lunge feeding effort across our range in body size suggests that all of the large whales studied are preparing for a lunge in similar ways. Field data (Cade et al., 2020) and hydrodynamic models (Potvin et al., 2009) suggest that the whales begin lunges at high speeds (3.5-5 m s-1) in order to overcome heightened drag during engulfment and krill-feeders usually move through the prey patch on momentum (Potvin et al., 2009).

Focusing more heavily on the relationship between mass-specific thrust generation and body size, our results diverge slightly from previous estimates. Fish (1998) determined that mass-specific thrust and body size have no relationship. Hill (1950) considered that for similar animals, the maximum power generated during a steady effort would increase not directly with the weight (W), but rather with W0.73. As a result, we expected that mass-specific thrust would decrease proportionately with increasing body size. Instead, we found that mass-specific thrust increases as body length increases (Fig. 4). This relationship could result from the higher oscillatory frequencies with larger body sizes that Gough et al. (2019) and our current study found in contrast to previous expectations of oscillatory frequency (Hill, 1950; Sato et al., 2007). For the relationship between oscillatory frequency and body length, Sato et al. (2007) found a more extreme allometric scaling exponent (approximately -1.0), whereas Gough et al. (2019) and our current study found an exponent of approximately -0.5, suggesting a less extreme decrease in oscillatory frequency with increasing body length.

*Drag Coefficient vs. Reynolds Number*

In comparison to our tagged animals, Hoerner’s R-100 airship model used for computational analysis did not include control surfaces (flippers or flukes). Instead, the approximated environment around the airship was determined using wind tunnel test data (Hoerner, 1965; Blevins, 1984). These modeled values suggest that for an Antarctic minke whale (~5m), the drag coefficients for fluking should be roughly three times higher than non-fluking and gliding. But the difference between these coefficients should increase for larger animals, culminating in a six-fold difference for a blue whale (~22m) (Fig. 6B). Other studies predicted similar increases in the drag coefficient, with Lighthill (1971) first noticing a discrepancy between the expected drag coefficient based on Hoerner’s model and the observed values for swimming fish, but his conclusions did not account for changing Reynolds numbers and were based upon animals swimming at Reynolds numbers of ~105 whereas large cetaceans are routinely swimming at values of ~107. Fish (1993) included a variety of species and groups and found higher drag coefficient values for swimming animals as compared to model estimates, but they did not find an increase with increasing Reynolds number like we have for larger cetaceans (Fig. 6C). Fish (1998) analyzed how the drag coefficient might vary with Reynolds number among four species of odontocetes and found that the drag coefficient should decrease with increasing Reynolds number.

For mysticetes, we found a negative relationship between the drag coefficient and the swimming speed as well as a positive relationship between the drag coefficient and body length (Fig. 5A-B). Reynolds number is affected by both the swimming speed and the body length of an animal, so we believe that the impact of body size between individuals is more extreme than the impact of swimming speed within individuals, resulting in a net positive impact of Reynolds number on drag coefficient (Fig. 5C). The effects of swimming speed on drag coefficient have been determined previously by Fish (1998) for a group of odontocetes, but ours is the first study that includes a large enough body size range to be able to parse out the effect of body size on both Reynolds number and drag coefficient.

*Froude Efficiency vs. Swimming Velocity*

Optimal locomotor speeds have been demonstrated for runners, flyers, and swimmers (e.g., Tucker, 1968; Webb, 1975; Hoyt & Taylor, 1981). The cost of transport (COT) has been used as the metabolic proxy that is inversely related to the Froude efficiency (Fish, 2000; Williams et al., 1993) and Yazdi et al. (1999) found that the minimum COT for the bottlenose dolphin (*Tursiops truncatus*) occurred at swimming speeds of 2.1 and 2.5 m s-1, respectively. These speeds coincided with the routine swimming speeds in wild populations. Similarly, gray whales (*Eschrichtius robustus*) and Antarctic minke whales cruise at the speed of the lowest COT (Sumich, 1983; Blix & Folkow, 1995). The minimum COT for the gray whale corresponded to the swimming velocity (2.0-2.5 m s-1) of migrations (Wyrick, 1954; Williamson, 1972; Sumich, 1983), which was similar to the velocity of maximum efficiency for whales in the present study. Antarctic minke whales, however, were determined to have a minimum COT at the maximum cruising velocity of 3.25 m s-1 (Blix & Folkow, 1995), which was 37% higher than the optimal velocity for maximum Froude efficiency in the present study. However, the velocity of maximum Froude efficiency (2.37 m s-1; current study) was within the range of swimming velocities (1.5-2.6 m s-1) for migrating Antarctic minke whales (Williamson, 1972). Correspondingly, the swimming velocity for maximum Froude efficiency of the blue whale (2.02 m s-1) and the humpback whale (2.00 m s-1) were within the ranges of migratory velocities of 1.5-3.1 m s-1 (Williamson, 1972) and 1.1-4.0 m s-1 (Chittleborough, 1953; Williamson, 1972), respectively.

Our results show that Froude efficiency increases rapidly below ~2 m s-1 and plateaus, which broadly agrees with the results from Fish (1998) for ontocetes. In contrast with their data, we found that Froude efficiency tapers off above ~5 m s-1 during lunge-associated swimming, but not routine swimming. This could be an artifact of our dataset having fewer lunge-associated tailbeats or could suggest a kinematic shift between the two swimming modes – such as a change in the amplitude of heave or angle of attack of the fluke – that is not being accounted for in our estimations. If the latter is true, the whales could be optimizing for another parameter, such as prey capture efficiency or momentum generation, in exchange for a lower Froude efficiency at high speeds.

A reduced Froude efficiency would increase the energetic expenditure when foraging. Such an increase would have consequences to the whale’s energy budget and limit dive duration and associated feeding efficiency (Acevedo-Guitteriez et al., 2002; Croll et al., 2005). Compared to the Balaenopteridae, the Balaenidae (i.e., right whales, bowhead whale) swim slowly through the water and maintain a steady speed during ram filter feeding (Burns et al., 1993; Simon et al., 2009). The foraging strategy of the rorquals places greater energetic demands on balaenopteridae than the balaenids (Potvin & Werth, 2016; Potvin & Werth, 2017).

*Froude Efficiency vs. Total Body Length (m)*

In this study, Froude efficiency relates to the amount of mechanical work the animal does to propel itself forward. Previous research has shown that Froude efficiency would remain constant or slightly increase with increasing body size (Fish, 1998). However, we found that Froude efficiency decreases with increasing body size among rorquals (See Fig. 7B). The mechanistic explanation of this finding is that larger individuals have a slightly increased thrust generation but a greatly increased drag coefficient (Figs. 4 and 6), thus resulting in a lower Froude efficiency, because more energy may be required to overcome drag and achieve equivalent locomotor performance.

Our analyses suggest that size is an important determinant of swimming performance in rorquals. Balaenopteridae exhibit a size range than spans an order of magnitude in body mass, from Antarctic minke whales to blue whales (Lockyer, 1976). The scale of these ocean giants necessitates the use of oscillatory lift-based swimming as an effective propulsive mechanism for high-speed swimming at high Reynolds numbers (Webb and De Buffrénil, 1990; Fish, 2020). Interestingly in parallel with the trend of maximum speed in which intermediate animals exhibited the highest performance with lower maximum speeds for small and large animals, it was found for whales that Froude efficiency, another locomotor performance variable, decreased above and below an optimal size (Hirt et al., 2017) (Fig. 7; Table 4).

*Conclusions*

Although the thrust power required to achieve maximum speed appears to be constrained at large body sizes, large rorquals may benefit from a low cost of transport (Williams, 1999; Gough et al., 2019). Accordingly, the high burst velocities and accelerations attained by the whales during lunges would incur reduced Froude efficiencies compared to routine and migratory velocities. The reduced Froude efficiency would place increased energetic demands during active foraging using a lunge feeding strategy for the balaenopterids. Our results are some of the first to quantify the fine-scale hydrodynamics that underlie these energetic differences between routine and energetically expensive swimming modes and include some of the largest absolute body sizes as well as a larger body size range than any previous study on swimming animals.

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**Competing Interests:**

We have no competing interests to report.

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**Tables/Figures/Legends**

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|  | | | | ***Kinematics*** | | | ***Morphometrics*** | | | | | | |
| ***Species*** | ***Number of Individuals*** | ***Swim Speed (Routine) (m s-1)*** | ***Oscillatory Frequency (Routine) (Hz)*** | | ***Swim Speed (Lunge) (m s-1)*** | ***Oscillatory Frequency (Lunge) (Hz)*** | | ***Total Length (m)*** | ***Wetted Surface Area (m2)*** | ***Body Mass (kg)*** | ***Chord Length (m)*** | ***Fluke Area (m2)*** |
| ***Humpback*** | 31 (30) | 2.15 ± 0.066 | 0.24 ± 0.006 | | 2.85 ± 0.100 | 0.34 ± 0.011 | | 11.09 ± 0.33 | 61.50 ± 1.80 | 20537.85 ± 1363.54 | 1.05 ± 0.03 | 3.14 ± 0.18 |
| ***Blue*** | 18 | 2.27 ± 0.063 | 0.19 ± 0.004 | | 3.05 ± 0.056 | 0.24 ± 0.004 | | 22.50 ± 0.32 | 152.50 ± 2.20 | 67301.16 ± 3172.36 | 1.29 ± 0.03 | 4.71 ± 0.18 |
| ***Antarctic*** ***Minke*** | 14 | 2.44 ± 0.053 | 0.40 ± 0.010 | | 2.96 ± 0.117 | 0.49 ± 0.008 | | 7.30 ± 0.34 | 25.54 ± 1.21 | 5528.91 ± 450.57 | 0.55 ± 0.03 | 0.77 ± 0.06 |
| ***Bryde’s*** | 2 | 1.76 ± 0.51 | 0.25 ± 0.005 | | 3.11 ± 0.629 | 0.42 ± 0.01 | | 12.04 ± 2.07 | 51.32 ± 16.39 | 11737.54 ± 5193.87 | 0.81 ± 0.13 | 1.97 ± 0.56 |
| ***Fin*** | 2 | 3.02 ± 0.125 | 0.25 ± 0.015 | | 3.61 ± 0.900 | 0.32 ± 0.018 | | 18.90 ± 0.43 | 109.90 ± 2.50 | 39515.13 ± 2330.65 | 1.07 ± 0.07 | 2.78 ± 0.35 |
| ***Sei*** | 1 | 2.23 | 0.22 | | 2.46 | 0.30 | | 16.62 | 92.71 | 27275.04 | 1.15 | 3.23 |

Table 1. Kinematic and morphometric variables used for modeling of hydrodynamic properties for all (n=63) individual whales in our dataset. Those with an asterisk were modeled using available data and methods in the literature. All values are given as the mean ± the standard error.

|  |  |  |  |
| --- | --- | --- | --- |
| ***Oscillatory Frequency (Hz) vs. Total Length (m)***  ***(Figure 3)*** | ***Linear equation*** | ***R2*** | ***P – value*** |
| Routine Effort Swimming | ŷ = -0.560x + 0.313 | 0.77 | <0.001 |
| Lunge-Associated Swimming | ŷ = -0.565x + 0.009 | 0.76 | <0.001 |
| ***Swim Speed (m s-1) vs. Total Length (m)***  ***(Figure 3)*** |  |  |  |
| Routine Effort Swimming | ŷ = -0.003x + 0.785 | 6.25\*10-5 | 0.949 |
| Lunge-Associated Swimming | ŷ = 0.078x + 0.873 | 0.04 | 0.098 |
| ***Mean Mass-Specific Thrust Power vs. Swim Speed (m s-1)***  **(Figure 4)** |  |  |  |
| Routine Effort Swimming | ŷ = 0.731x – 2.764 | 0.40 | < 0.001 |
| Lunge-Associated Swimming | ŷ = 0.630x – 2.138 | 0.49 | < 0.001 |
|  |  |  |  |
| ***Mean Mass-Specific Thrust Power vs. Total Length (Figure 4)*** |  |  |  |
| Routine Effort Swimming | ŷ = 0.033x – 1.594 | 0.23 | < 0.001 |
| Lunge-Associated Swimming | ŷ = 0.031x – 0.693 | 0.16 | < 0.001 |
|  |  |  |  |
| ***Drag Coefficient vs. Swim Speed (m s-1)***  ***(Figure 5)*** |  |  |  |
| Routine Effort Swimming | ŷ = -0.613x – 2.617 | 0.12 | 0.004 |
| Lunge-Associated Swimming | ŷ = -0.270x – 3.148 | 0.06 | 0.042 |
| ***Drag Coefficient vs. Total Length (m)***  ***(Figure 5)*** |  |  |  |
| Routine Effort Swimming | ŷ = 0.082x – 5.080 | 0.64 | <0.001 |
| Lunge-Associated Swimming | ŷ = 0.058x – 4.737 | 0.41 | <0.001 |
| ***Drag Coefficient vs. Reynolds Number***  ***(Figure 5)*** |  |  |  |
| Routine Effort Swimming | ŷ = 2.92\*10-8x – 4.805 | 0.41 | <0.001 |
| Lunge-Associated Swimming | ŷ = 1.36\*10-8x – 4.476 | 0.23 | <0.001 |
| ***Froude Efficiency vs. Total Length (m)***  ***(Figure 6)*** |  |  |  |
| Routine Effort Swimming | ŷ = -0.004x – 0.049 | 0.68 | < 0.001 |
| Lunge-Associated Swimming | ŷ = -0.003x – 0.068 | 0.47 | < 0.001 |
|  |  |  |  |

Table 2. This table contains equations, estimates, R2 values, and p values from generalized linear mixed models for sequential figures 3-7 and 9.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ***Hydrodynamic Calculations*** | | | |
| ***Species*** | ***Mass-Specific Thrust Power (Watts kg-1)*** | ***Drag Coefficient*** | ***Reynolds Number*** | ***Froude Efficiency*** |
| ***Humpback*** | 0.30 ± 0.023 | 0.02 ± 0.002 | 2.29 x 107  ±  9.9 x 105 | 0.908 ± 0.003 |
| ***Blue*** | 0.48 ± 0.025 | 0.04 ± 0.006 | 4.89 x 107  ±  1.3 x 106 | 0.860 ± 0.006 |
| ***Antarctic*** ***Minke*** | 0.36 ± 0.028 | 0.01 ± 0.001 | 1.71 x 107  ±  9.9 x 105 | 0.920 ± 0.004 |
| ***Bryde’s*** | 0.50 ± 0.213 | 0.04 ± 0.012 | 2.13 x 107  ±  9.4 x 106 | 0.868 ± 0.022 |
| ***Fin*** | 0.74 ± 0.130 | 0.02 ± 0.007 | 5.46 x 107  ±  1.0 x 106 | 0.889 ± 0.018 |
| ***Sei*** | 0.51 | 0.03 | 3.54 x 107 | 0.876 |

Table 3. Results from hydrodynamic and morphometric calculations for all individuals (n=63) from each species. All values are given as the mean of all tailbeats in a deployment ± the standard error. The drag coefficient, Reynolds number, and Froude efficiency are dimensionless. The mean fluke area is shown in m2 and the chord length and total lengthare shown in meters.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Species*** | ***Swim Speed (m s-1) or (bl s-1)\**** | ***Total Length (m)*** | ***Froude Efficiency*** | ***Source(s)*** |
| ***Homo sapien***  *Human (Female)* | 0.95 | 2.38 | 0.29 | von Loebbecke et al., 2009 |
| ***Ondatra zibethicus***  *Muskrat* | 0.75 | 0.44 | 0.33 | Fish, 1984 |
| ***Pterophyllum eimekei***  *Freshwater Angelfish* | 0.04 | 0.08 | 0.16 | Blake, 1979; Blake, 1980 |
| ***Danio rerio***  *Zebra Danio* | Multiple | 0.0315 | 0.80 | McCutchen, 1975 |
| ***Cymatogaster aggregata***  *Shiner Perch* | 0.57 | 0.143 | 0.65 | Webb, 1975 |
| ***Oncorhynchus mykiss***  *Rainbow Trout* | Ucrit | 0.293 | 0.75 | Webb, 1975 |
| ***Euthynnus affinis***  *Mackerel Tuna (Kawakawa)* | 1.52 | 0.40 | 0.90 | Magnuson, 1978 |
| ***Pusa hispida***  *Ringed Seal* | 0.75 | 1.03 | 0.88 | Fish et al., 1988 |
| ***Pagophilus groenlandicus***  *Harp Seal* | 1.04 | 1.43 | 0.87 | Fish et al., 1988 |
| ***Trichechus manatus***  *American Manatee* | 0.30\* | 3.23 | 0.83 | Kojeszewski and Fish, 2007 |
| ***Delphinapterus leucas***  *Beluga Whale* | 3.00 | 3.64 | 0.84 | Fish 1998 |
| ***Lagenorhynchus obliquidens***  *Pacific White-Sided Dolphin* | 5.30 | 2.00 | 0.89 | Webb, 1975; Yates, 1983; Blickhan and Cheng, 1994 |
| ***Orcinus orca***  *Killer Whale* | 6.50 | 4.74 | 0.88 | Fish, 1998 |
| ***Pseudorca crassidens***  *False Killer Whale* | 3.80 | 3.75 | 0.90 | Fish, 1998 |
| ***Sotalia guianensis***  *Guiana Dolphin* | 2.40 | 1.90 | 0.83 | Blickhan and Cheng, 1994 |
| ***Tursiops truncatus***  *Common Bottlenose Dolphin* | 2.401, 3.802 | 2.501, 2.612 | 0.781, 0.862 | Blickhan and Cheng, 19941; Fish, 19982 |
| ***Balaenoptera physalus***  *Fin Whale* | 3.02 ± 0.125 (Routine Effort Swimming) | 18.90 ± 0.43 | 0.899 ± 0.018 | Current Study |
| ***Balaenoptera bonaerensis***  *Antarctic Minke Whale* | 2.44 ± 0.053 (Routine Effort Swimming) | 7.30 ± 0.34 | 0.920 ± 0.004 | Current Study |
| ***Megaptera Novaeangliae***  *Humpback Whale* | 2.15 ± 0.066 (Routine Effort Swimming) | 11.09 ± 0.33 | 0.908 ± 0.003 | Current Study |
| ***Balaenoptera musculus***  *Blue Whale* | 2.27 ± 0.063 (Routine Effort Swimming) | 22.50 ± 0.32 | 0.860 ± 0.006 | Current Study |
| ***Balaenoptera borealis***  *Sei Whale* | 2.23 (Routine Effort Swimming) | 16.62 | 0.876 | Current Study |
| ***Balaenoptera brydei***  *Bryde’s Whale* | 1.76 ± 0.51 (Routine Effort Swimming) | 12.04 ± 2.07 | 0.868 ± 0.022 | Current Study |

Table 4. Froude efficiency and metadata collected from various sources for the creation of figure 8.

Figure 1. Adaptation from Shadwick (2005) showing the forces acting on the tail of a thunniform swimmer such as a blue whale during active oscillatory fluking of the tail. The heaving motion of the tail creates a pressure imbalance between the top and bottom faces of the fluke that results in the generation of a lift force perpendicular to the path of the flukes and a thrust force in the forward direction of travel of the animal.

Figure 2. Representative UAS drone image of a humpback whale showing the morphometric measurements taken from each animal. The white line corresponds to the total length (in meters) from the tip of the lower jaw to the caudal midpoint of the tail. The chord length of the fluke (in meters) is denoted by the red line running from the cranial insertion of the fluke onto the peduncle to the caudal midpoint of the tail. The purple shaded region corresponds to the tail area (in m2) comprising the entirety of the flukes and the peduncle region caudal to the cranial fluke insertions. Make lines thicker.

Figure 3. Linear regressions showing the log10 of total body length (m) versus the A) oscillatory frequency (Hz) and B) swim speed (m s-1) for both routine swimming (solid line) and lunge-associated swimming (dashed line). Each point corresponds to the mean value for a single individual whale and a single swimming mode (● circle: routine; ▲ triangle: lunge-associated).

Figure 4. Linear regressions showing A) swim speed (m s-1) and B) total body length (m) versus mass-specific thrust power output (W kg-1) for both routine swimming (solid line) and lunge-associated swimming (dashed line). Each point corresponds to the mean value for a single individual whale and a single swimming mode (● circle: routine; ▲ triangle: lunge-associated). One point was removed above the visual range of each figure (Bryde’s whale: mean swim speed = 3.74 m s-1; total body length = 14.1 m; mean mass-specific thrust power output = 2.60 Watts kg-1).

Figure 5. Linear regressions showing A) swim speed (m s-1), B) total body length (m), and C) Reynolds number (dimensionless) versus the drag coefficient (dimensionless) for both routine swimming (solid line) and lunge-associated swimming (dotted line). Each point corresponds to the mean value for a single individual whale and a single swimming mode (● circle: routine; ▲ triangle: lunge-associated). One point was removed above the visual range of each figure (blue whale: mean swim speed = 1.68 m s-1; total body length = 24.1 m; mean Reynolds number = 3.9e+07; mean drag coefficient = 0.131).. Dot-dash lines shown in C) are linear regressions of Reynolds number versus drag coefficient for simple rigid-body model comparison using equations derived from Hoerner (1965). Illustration shows a swimming blue whale and image shows an R100 rigid-body as visual representations of the data shown in C).

Figure 6. Curved fit lines showing A) swim speed (m s-1) and linear regressions showing B) total body length (m) versus propulsive efficiency (dimensionless) for both routine swimming (solid line) and lunge-associated swimming (dotted line). Curved fit lines are shown in A) to better illustrate the effects of maximal performance on Froude efficiency, and each point in B) corresponds to the mean value for a single individual whale and a single swimming mode (● circle: routine; ▲ triangle: lunge-associated). Plots along x-axis of A) shows the density of swim speeds for routine (solid border) and lunge-associated swimming (dotted border). One point was removed above the visual range of each figure (Bryde’s whale: mean swim speed = 3.74 m s-1; total body length = 14.1 m; mean mass-specific thrust power output = 2.60 Watts kg-1).

Figure 7. Propulsive efficiency versus total body length (m) for species from different morphological and taxonomic groups and use different swimming modes (● circle: drag-based paddling; ▲ triangle: undulatory swimming; ■ square: oscillatory swimming). The mysticete cetaceans are the mean species-level data from our present study. Silhouettes correspond to each group by rough position and color.